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ABSTRACT

One of the five goals proposed in the National Council of Teachers of Mathematics' (NCTM) "Curriculum and Evaluation Standards for School Mathematics" is for students to become "mathematical problem solvers." To achieve this goal, the NCTM proposes fundamental changes in both the content of mathematics curricula and the pedagogy of the mathematics classroom that outline a broad role for problem solving. Textbooks and other published curricula are believed to play a significant role in the classroom instruction of mathematical problem solving. Described and compared are the conceptions of problem solving found in one commonly used elementary mathematics textbook ("Addison-Wesley Mathematics") and three distinctive elementary mathematics curricula ("Real Math", "Comprehensive School Mathematics Program", and "Math in Stride"). Although in each, problem solving was professed to be of central importance to the development of mathematical understanding and competence, how problem solving is defined and taught varied amongst the four published curricula. The different ways that problem solving is defined, formulated, and taught are explored, and the pedagogical and epistemological assumptions that underlie these differences are also discussed. The four curricula are compared on various perspectives of problem solving, the levels at which it was incorporated into the curriculum and how the authors of each curriculum viewed "mathematics as problem solving." (14 references) (MDH)

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CONCEPTIONS OF PROBLEM SOLVING IN COMMONLY USED AND DISTINCTIVE ELEMENTARY MATHEMATICS CURRICULA

Janine Remillard

Center for the Learning and Teaching of Elementary Subjects

Institute for Research on Teaching

College of Education

Michigan State University

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Janine Remillard

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Center for the Learning and Teaching of Elementary Subjects

The Center for the Learning and Teaching of Elementary Subjects was awarded to Michigan State University in 1987 after a nationwide competition. Funded by the Office of Educational Research and Improvement, U.S. Department of Education, the Elementary Subjects Center is a major project housed in the Institute for Research on Teaching (IRT). The program focuses on conceptual understanding, higher order thinking, and problem solving in elementary school teaching of mathematics, science, social studies, literature, and the arts. Center researchers are identifying exemplary curriculum, instruction, and evaluation practices in the teaching of these school subjects; studying these practices to build new hypotheses about how the effectiveness of elementary schools can be improved; testing these hypotheses through school-based research; and making specific recommendations for the improvement of school policies, instructional materials, assessment procedures, and teaching practices. Research questions include, What content should be taught when teaching these subjects for understanding and use of knowledge? How do teachers concentrate their teaching to use their limited resources best? and In what ways is good teaching subject matter-specific?

The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

The findings of Center research are published by the IRT in the Elementary Subjects Center Series. Information about the Center is included in the IRT Communication Quarterly (a newsletter for practitioners) and in lists and catalogs of IRT publications. For more information, to receive a list or catalog, or to be placed on the IRT mailing list to receive the newsletter, please write to the Editor, Institute for Research on Teaching, 252 Erickson Hall, Michigan State University, East Lansing, Michigan 48824-1034.

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Abstract

Phase II of the research on the teaching and learning of higher order thinking and conceptual understanding in the elementary subjects, undertaken by the Center for the Learning and Teaching of Elementary Subjects, includes description and analysis of commonly used and distinctive published curricula in each of the five subject areas. This paper describes and compares the conceptions of problem solving found in one commonly used elementary mathematics textbook (*Addison-Wesley Mathematics*) and three distinctive elementary mathematics curricula (*Real Math, Comprehensive School Mathematics Program,* and *Math in Stride*). In each, problem solving was professed to be of central importance to the development of mathematical understanding and competence. Nevertheless, how problem solving is defined and taught varies considerably amongst the various published curricula. In this paper, the different ways problem solving is defined, formulated, and taught are explored. The pedagogical and epistemological assumptions that underlie these differences are also discussed.

CONCEPTIONS OF PROBLEM SOLVING IN COMMONLY USED AND DISTINCTIVE ELEMENTARY MATHEMATICS CURRICULA[1]

Janine Remillard[2]

"Becoming a mathematical problem solver" is one of the five goals proposed in the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989). It is considered essential to productive citizenship in a technological, information-oriented society. To achieve this goal, NCTM proposes fundamental changes in both the content of mathematics curricula and the pedagogy of the mathematics classroom. In the Standards, NCTM outlines a broad role for problem solving. The development of a problem-solving approach to learning and doing mathematics is proposed as both a curricular goal and process by which other curricular goals should be achieved. Seen as the "central focus of the mathematics curriculum," "a method of inquiry and application," and an approach to learning mathematical content rather than a strand or topic that can be isolated, it is intended to "permeate the entire program" (p. 23), providing the context in which skills and concepts are learned and applied. Problem-solving opportunities, NCTM contends, will motivate students to develop concepts and skills. Through these opportunities students can experience the "power and usefulness of mathematics." Thus, the term "mathematics as problem solving," found in the Standards, depicts problems solving as an authentic mathematical activity through which students should learn that mathematics is powerful, useful, and interesting. Furthermore, it should provide opportunities and foster motivation to develop, practice, and apply mathematical knowledge.

Textbooks and other published curricula are believed to play a significant role in classroom instruction (Freeman & Porter, 1989). This study analyzed the content, pedagogical

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[1]This paper was presented at the annual meeting of the American Educational Research Association in Boston, April 1990.

[2]Janine Remillard, a doctoral candidate in teacher education at Michigan State University, is a research assistant with the Center for the Learning and Teaching of Elementary Subjects. The author gratefully acknowledges Magdalene Lampert, Penelope Peterson, Ralph Putnam, and Deborah Lowenberg Ball for their advice in the conceptualization of this work and for their helpful comments on earlier drafts.

vision and epistemological assumptions of four elementary mathematics curricula. In order to capture the diverse range of curricular resources, and the conceptions of mathematics embedded in them, that could potentially influence practice, three of the four curricula were distinctive, aiming to provide a distinct epistemological and pedagogical alternative to the commonly used curricula. Whereas Addison-Wesley Mathematics, published in 1987, is more widely used, the three distinctive curricula, Real Math, published by Open Court in 1987, Comprehensive School Mathematics Program (CSMP), published by CEMREL in 1978, and Math in Stride, published by Addison-Wesley in 1988, propose views of mathematics and mathematics teaching and learning that break from that traditionally found in American schools. All four programs are designed to be used in typical elementary classrooms from first to sixth grade. Together they present four varying portraits of the nature of mathematical knowledge, knowing, and learning.

Overwhelmingly evident in each of the analyzed curricula was attention to developing students' problem-solving abilities. Nevertheless, there seems to be little consensus about what it is and why and how it should be part of the curriculum. The authors of each curriculum acknowledge that problem solving is crucial in mathematics and claim that their program emphasizes, teaches, or engages students in problem solving. But their perspectives on the particulars of problem posing and use vary substantially. It is these varying perspectives that I examine in this paper, comparing the ways four elementary mathematics curricula define, formulate, and use problems. For an alternative perspective, I will look to the vision of problem solving proposed by the NCTM Standard, "Mathematics as Problem Solving."

From Rhetoric to Substance

It is apparent that the curriculum authors have caught on, at least, to a "rhetoric of problem solving" (Schoenfeld, 1989, p. 348) . Each of the analyzed curricula professes to hold problem solving as central to learning mathematics. In Addison-Wesley Mathematics, problem solving is billed as being "for the decision-makers of the future" (p. T8). In a statement under the heading "How Addison-Wesley Mathematics Puts Research into Practice," the authors claim that "problem

solving is the key to understanding why computational skills are needed. . . . [It] should be an integral part of any computational skills lesson" (p. T26). The authors of Real Math provide a detailed statement of their position on problem solving:

> The main purpose of teaching mathematics is to develop the ability to use mathematical thought to solve problems. . . . Problems in real life do not come so neatly packaged. Nor can they be taught by providing word problems that can all be solved by applying whatever algorithm has just been presented. Children must learn that thinking is necessary in mathematics and that thinking can save work and make it possible to solve problems. (p. xxi)

They go on to describe the various type of problem solving found in their text. Problem solving is also professed to be central to CSMP. Its development was based on an assumption that problem solving was an area of the curriculum which had been undertreated as it was difficult to teach and teachers were underprepared to teach it. One belief that guided the developers of the program is that "mathematically rich problem solving activities should be prominent and should generate topics, guide content, sequencing, and provide computation practice" (Herbert, 1984, p. 31). The authors of Math in Stride are the least explicit about their reasons for including problem solving, yet they claim to have placed major emphasis on it in their curriculum. Claiming that problem solving is explored through data organization, spatial relationships and numerical relationships, the authors explain that "through the problem-solving design students are given opportunities to develop ways to think creatively and solve problems . . . and pose new problems based on their own learning" (p. vii). In these analyses of these curricula, I look beyond these rhetorical statements to the actual substance of the programs and aim to portray the view of mathematical problem solving reflected in the actual content of each curriculum.

Perspectives on Problems and Mathematical Problem Solving

The lack of consensus on what makes a good problem and what is included in mathematical problem solving evident in the analyzed curricula reflects the diverse positions in the mathematics education community. There is some agreement that mathematical problems must be truly problematic in nature, but various criteria have been used to determine what this means.

These include the form of the problem, its source, and the solution process. Thus, the traditional story problem--a one- or two-sentence story followed by a question--is criticized as being artificial and unproblematic, a dressed-up computation exercise rather than a real-life application of mathematics. Focusing on the form and solution process, Schoenfeld (1985) calls most story problems exercises, rather than actual problems, insisting that problems that are truly problematic do not have clear or single solution paths. But, Yackel, Cobb, and Wood (1988) argue that some computational exercises can be considered problems. Judging problems by the primary process of solving the problem, they insist that computational exercises for which students do not have a readily-accessible method of approach can be truly problematic. Putnam, Lampert, and Peterson (1989) also consider the source of the problem to be relevant criteria, arguing that authentic problems should arise from patterns and irregularities observed by the students that cause disequilibrium or curiosity. So, what makes a good problem is partly determined by the students' observations, interest, and previous experience.

The discussion of "mathematics as problem solving" in the Standards focuses on the form of the problem and the solution process. Examples of problems and how students might engage them are provided, but less attention is given to the source of these problems. There is little explicit guidance for problem formulation, selection, and posing but a clear differentiation is made between traditional word problems, "which provide contexts for using particular formulas or algorithms but do not offer opportunities for true problem solving" (p. 76), and nonroutine problems. The latter are said to be often messy and more complicated, often having multiple solutions. Other criteria for selecting and formulating worthwhile problems must otherwise be distilled from the goals and descriptions of problem-solving activities. These present situations in which the task includes collaborative sense making of the problem, developing strategies for solving it, recording data, and determining whether or not the solution is reasonable and complete.

The Standards suggest that a classroom oriented toward problem solving engages students in solving, through the development and application of strategies, a wide variety of

problems, verifying and interpreting their results, investigating mathematical content through problem-solving experiences and communicating their thinking and strategies to others. Topics, such as geometry, permutations, and functions can be explored through "thought-provoking questions, speculations, investigations and explorations" (p. 23). Additionally, a vital problem-solving activity proposed by NCTM is problem formulation--formulating problems from both everyday and mathematical situations. Through carefully chosen problem-solving experiences, students can work directly with mathematical content, applying it to problem situations.

Mathematical problem solving is thought to help students develop strategies that will equip them to solve problems in their lives. This is not to say that the Standards equates problem solving with real-life applications or limits the scope of appropriate problem activities to actual life encounters. Instead, it acknowledges that mathematical ideas also pose puzzling and problematic situations which present worthwhile problems.

What follows is a description of the problem solving found in each of the four curricula analyzed, including how problems are defined and formulated, their place and purpose in the curriculum, and how problem-solving strategies are developed. This is followed by a comparative look at the four curricula.

Addison-Wesley Mathematics

The authors of Addison-Wesley's commonly used curriculum have heeded the current calls from the mathematics education community for increased emphasis on problem solving and realistic applications of mathematics. In addition to building problems into the daily lessons, they have devoted entire lessons to teaching problem-solving strategies and approaches. These efforts are significant. Nevertheless, a close look at how the problems are formulated and used, in contrast to the vision presented in the Standards, suggests that these intentions fall short of their goals.

Dressed-up algorithms. The problems presented in Addison-Wesley Mathematics are predominantly tightly controlled, single-step word problems, containing straightforward

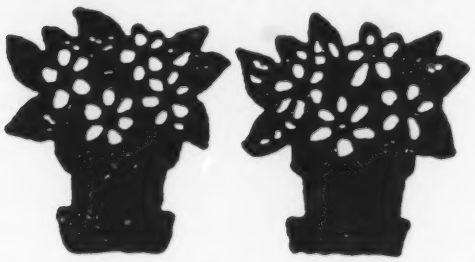
computational exercises within real-life settings. Found throughout the text and on chapter tests, these problems require the solver to distill the "story" into an algorithmic exercise and solve it computationally. They tend to call for a single operation and have a single answer. The fact that they occur mainly at the end of each chapter or as the last two or three items on a page suggests that their purposes are to provide practice with the application of specific algorithmic skills taught in the preceding lesson and to demonstrate ways that these skills can be used. On some occasions the students are asked to write the algorithm next to the story problem which further emphasizes that each problem can be distilled into a computational algorithm. (See Figure 1.) Because each problem presented in the curriculum generally contains a single algorithm, mastery of the particular algorithm is a necessary prerequisite to solving the problem.

What is missing from the emphasis on algorithmic mastery is the need to make sense of the situation and the problem it poses. This is particularly salient in instances in cases where the problem solving has been unduly simplified. By removing much of the complexity and uncertainty inherent in the actual situation and in the sort of problems recommended in the Standards, the authors place greater emphasis on getting the right answer to an artificial problem than on understanding the real problem. Key words or phrases, such as "in all," "left," or "all together" pervade most of the problems so that the student can select the correct operation without necessarily understanding the question. The chapter organization also makes the necessary operation or procedure to solve a problem obvious. Division problems with 1-digit divisors, for example, are grouped together in one chapter and each lesson is grouped by the number of digits in the quotient.

Because the applications are used to practice specific skills, rarely does a student need to determine the operation or procedure to use. Even exercises that require second graders to determine whether or not regrouping is necessary become simplified when all the exercises that do not require regrouping are placed together at the beginning of the page. (See Figure 2.)

Application. Learning to apply mathematics in one's daily life is certainly a realistic goal of elementary mathematics. NCTM emphasizes that using problem situations from the students'

8. Jane has 2 plants. Each plant has 4 flowers. How many flowers in all?



8 flowers

$$\begin{array}{r} 4 \\ \times 2 \\ \hline 8 \end{array}$$

Figure 1. Emphasis on algorithms in second grade Addison-Wesley Mathematics (Addison-Wesley, 1987, Teachers' Edition, Book 2, p. 309).

Name _____

Subtract.

1.	47 −23 24	32 −11 21	58 − 6 52	74 −50 24	86 −56 30	56 −36 20
2.	2 12 ~~32~~ −14 18	3 11 ~~41~~ −25 16	1 15 ~~25~~ −17 8	4 14 ~~54~~ −27 27	5 12 ~~62~~ −48 14	4 10 ~~50~~ −18 32
3.	3 16 ~~46~~ −38 8	1 16 ~~26~~ − 8 18	6 11 ~~71~~ −24 47	7 13 ~~83~~ −78 5	6 10 ~~70~~ −53 17	7 12 ~~82~~ − 8 74
4.	6 11 ~~71~~ −26 45	3 16 ~~46~~ −29 17	3 10 ~~40~~ −12 28	2 15 ~~35~~ −19 16	6 12 ~~72~~ −18 54	4 13 ~~53~~ −26 27

5. Solve.

Carla had 73 baseball cards.
She sold 55 baseball cards.
How many cards did she have left?

18 cards

6 13
~~73~~
−55
18

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Figure 2. Sequencing of subtraction exercises on second grade Addison-Wesley Mathematics (Addison-Wesley, 1987, Teachers' Edition, Book 2, p. 221).

actual experiences helps them to see the relevance of mathematics to their lives. Addison-Wesley has incorporated applications into its curriculum, but rather than applying mathematical thinking and problem solving to potentially real situations, rote skills and procedures are being applied to situations from real-life contexts that do not necessarily fit well with the problem.

The term "application" is used throughout the curriculum in reference to word problems. The implication is that these problems provide opportunities for students to apply mathematical skills to real-world situations. The mathematical task of each story situation, however, is so tightly tied to the skill being taught in the particular chapter that the students are often asked to find information that would be irrelevant, unrealistic or already known if the situation were real. The following is an example problem from the fifth-grade curriculum, posed after several lessons on adding and subtracting mixed numbers:

> The old school high jump record was 6 feet 5 1/4 inches. The new record was 6 3/8 inches higher. What was the new record? (Grade 5, p. 259).

It is difficult to imagine that a new school record would be announced in terms of how much it exceeds the previous record, expecting the spectators and athletes to perform the addition and determine the new record. Would the height of the high jump bar not have been known before the athlete approached the bar and jumped it successfully? This situation is indeed drawn from real life, but the mathematics imbedded in it is tangential to the situation.

Many application problems in Addison-Wesley Mathematics do present situations that a child might actually confront with related and realistic mathematical tasks, such as determining if she has enough money to buy a 49-cent toy car, given that she has a half dollar and three pennies (Grade 2, p. 144). Another problem asks for the number of rolls of wall paper needed to cover a $12m^2$ wall, given that one roll will cover $5m^2$ (Grade 5, p. 199). This latter problem requires that the student truly make sense of this situation and consider the number of full rolls necessary to completely cover the wall. A student formulating such a problem into an algorithm 12÷5 is likely to arrive at an answer of 2 remainder 2 or 2 2/5, rather than determining that 3 full rolls will be

necessary to cover the wall completely. (Just in case, the authors remind the student that full rolls must be purchased.) Furthermore, wallpaper is generally purchased in rolls that specify the total area that can be covered. While being somewhat realistic situations they are posed in forms that require the solver to confront only the algorithmic portion of the problem. In real life, the solver would need to determine the area of the wall, among other things, before calculating the number of rolls of wallpaper to buy. Those who have wallpapered know that the width of the paper and the actual dimensions of the room are likely to affect the number of rolls needed. As stated, this problem does not require that the students explore ideas related to area. Rather it is merely a division problem, in which the units of the dividend and divisor are ostensibly irrelevant.

Problem-solving strategies. The development of strategies useful in solving problems is another example of Addison-Wesley's response to current calls for helping students to make sense of and think mathematically to solve problems they face in their lives. Throughout history, strategies or heuristics have been used to solve problems (Polya, 1957). These useful mental operations are what Polya saw as crucial components of mathematical thinking that are not part of formal mathematics. More recent research on problem-solving abilities has indicated that problem-solving strategies increase success in problem solving (Schoenfeld, 1985). Addison-Wesley's authors have recognized the importance of using strategies by attending to strategy development. But the lessons are not integrated into the entire curriculum. Thus, it is unclear whether problem-solving strategies are seen as applicable in all problem situations, or only in those found in particular lessons.

Addison-Wesley emphasizes the development of problem-solving strategies under completely separate contexts than the above-cited word problems. Isolated lessons are devoted to strategies such as gathering data from a chart, eliminating unnecessary information, and working backwards. Additionally, a five-step approach to problem solving is taught: (1) Understand the question; (2) find the needed data; (3) plan what to do; (4) find the answer; (5) check back. Practice using these steps is provided in problem-solving lessons. These steps and strategies, however, are rarely used with the application problems mentioned above or found on

chapter tests. The problems in the lessons specific to problem solving are like the nonroutine problems discussed in the Standards; that is, they often have multiple steps, extra information, and require the student to understand the situation and the question being asked. This is particularly true at the fourth- through sixth-grade levels. Some lessons focus on estimating and determining whether or not an answer is reasonable, but this is done by prescription: There are rigid steps to be followed, each has one right answer--even the estimation problems--and the lessons are designed to develop specific strategies or approaches. Students are not given practice using strategies in conjunction with one another or determining which strategy to use.

There are some problems with the possibility of multiple solutions and strategies. In these cases there is "no right answer." This is indicated by the statement in the teacher's guide, "Answers will vary." The implication is that any answer is right. An example of a problem with no right answer, shown in Figure 3, also illustrates how the text controls the problem task by walking the students through the steps of the solution, rather than allowing them to determine how to solve it. Note that the problem is broken into subquestions and the student is guided through each step. Exact answers are provided in the teacher's guide to these subquestions, yet it is expected that answers to the question of what time to leave will vary. The information in the margin encourages the teacher to "emphasize that decisions on problems such as these may vary, and that there is no right answer" (Grade 5, p. 213). The teacher is not encouraged to help students distinguish between realistic or reasonable answers and those that are ridiculous, or to have students provide reasons for their answers. The information in the teacher's guide could be interpreted as indicating that any answer is right whether or not it makes sense.

This is one of the more complex problems found in Addison Wesley Mathematics. Generally, opportunities for problem solving are restricted to single-step word problems. The authors do attend to the development of problem-solving strategies and approaches in sections devoted specifically to problem solving. It is not generally encouraged that these be applied to the word problems found in the specific skill sections of the text. This separation of strategies and problems may undermine the authors' intention of teaching students to be "the decision-makers

of the future" (p. T8). One interpretation could be that all problems have only quantifiable answers and can be formulated into one or several algorithms and solved computationally. Thus, in Addison-Wesley Mathematics, a gap exists between making sense and solving problems. In Real Math the authors have recognized this gap and attempted to fill it.

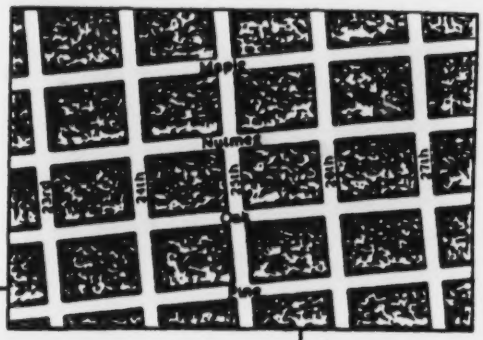
Real Math

In appearance, Real Math, published by Open Court, seems similar to commonly used curricula, such as Addison-Wesley Mathematics. A close look at the content treatment and specific pedagogical suggestions, however, reveals its distinctive nature. The authors of Real Math describe their program as aiming to make mathematics "real" and useful to children, rather than "obscure" and "artificial." This goal is reflected in their attempt to place learning mathematics within contexts that are real to children. The authors include problems that are playful and fantastic, as well as lifelike, arguing that these contexts are equally inviting to children. The problem-solving situations presented in the texts vary from simple word problems to complex, divergent story situations, yet even the word problems are less contrived and more problematic than those in Addison-Wesley's text. The authors have attempted to use problems in which the mathematical tasks are relevant to the problem situation which contains it.

There are three distinct types of problems in Real Math: word problems, Thinking Stories, and discussion questions. In each case, the problems are set in a variety of life situations which, while not always realistic, require making sense of the situation and applying mathematical algorithms or processes as part of the solution.

Problem-solving strategies. The authors emphasize the development of problem approaches and solution strategies. Unlike Addison-Wesley, however, through working with problems and thinking through the questions in the text, students are expected to develop the ability to mathematize situations as means of making sense of them and translating them into solvable problems. The problems are designed and posed in ways that necessitate strategizing and sense making. The authors have resisted simplifying strategy techniques, such as the use of

Applied Problem Solving



You and your friend Sandy want to take a bus downtown to a movie. The movie starts at 1:00 p.m. What time should you leave home in order to get to the movie on time?

Some Things to Consider

- You live at the corner of 23rd and Maple. Sandy lives at the corner of 26th and Nutmeg.
- You will first walk to Sandy's house; then the two of you will walk to the bus stop.
- You will catch the bus at 26th and Pine, travel on Pine, and get off the bus at 72nd and Pine.
- A bus comes every 10 to 15 minutes. It takes about 1 minute for a bus to go 1 block.
- You will walk from the bus stop at 72nd and Pine to the movie theater at 73rd and Oak.
- It takes you about 4 minutes to walk 1 block.

Some Questions to Answer

1. How long will it take you to walk to Sandy's house and from her house to the bus stop? 16 min to Sandy's; 8 min to bus
2. How long will it take the two of you to walk to the theater? 8 min
3. How much time should you plan to wait for the bus? 15 min
   How much time should you plan to spend on the bus? 46 min
4. How much extra time should you allow so that you are sure you will be on time? Answers will vary.

What Is Your Decision?

What time will you need to leave your home in order to get to the movie on time? Answers will vary.

Figure 3. Sample fifth-grade problem from Addison-Wesley Mathematics (Addison-Wesley, 1987, Teachers' Edition, Book 5, p. 213).

key words (e.g., "all together" to indicate additions, "left" to indicate subtraction) that indicate the necessary operation method. They explain that "problems in real life do not come so neatly packaged" (Open Court, Grade 5, p. xxi). Furthermore, there is an evident attempt, particularly in the fourth through sixth grade, to avoid following the practice of a specific operation with a homogeneous set of word problems that require the same operation, as found in Addison-Wesley.

Word problems. Word problems found consistently throughout Real Math and in the unit tests are brief stories that can be rephrased in the form of one or several symbolic statements. There are several distinctions in their presentation and the contexts in which they are presented which require sense making on the part of the solver, making them more problematic than those found in Addison-Wesley. While these problems are used to practice the applications of computational skills previously developed, care has been taken to avoid presenting a set of word problems that require the same operation. Consecutive problems are often related mathematically or use the same situation, but as a set they provide mixed practice, so that the student must understand the problem situation to determine which operation to use on which numbers. For example, the last two story problems in the 27th lesson of the fifth-grade text tell about Old Town School which has 161 students grouped equally into seven classes. The student is asked to tell how many students will be in each class. While the partitive method of division is the necessary operation in this first problem--161 students must be divided into seven equal groups-- subtraction is required to find the answer to the next, which asks for the number of total students once seven move away. Another lesson in which students have worked with all four operations concludes with four word problems. Each involves work with decimals, but various operation are required and the questions are framed, so that students must first determine what is being asked and how one might find the answer before translating it into an appropriate algorithmic form:

41. Dennis has $1.00. Can he buy 5 bananas that cost 17 cents each?

42. Alex bought a shirt that costs $13.95 plus $0.84 tax. He gave the clerk a $20 bill. How much change should he get back?

43. Lia is buying a football that costs $7.98. The tax comes to $0.48. How much will she have to pay?

44. Brenda jogs everyday from her house to the park and back. It is 1.2 kilometers from her house to the park.

    a. How far does Brenda jog in one day?
    b. How far does she jog in two days?
    c. How far does Brenda jog in 10 days? (Real Math, Grade 5, p. 115)

Although the story situations in the word problems found throughout the first- through third-grade texts are less complex and detailed, the problems retain their validity, presenting events similar to real experiences that are likely to evoke thought and some strategizing by that age student. The following is an example from the third-grade text that presents the student with a situation in which regrouping is needed:

Mrs. Taylor has one $100 bill and two $10 bills. She must pay Mr. Olson $30. (The picture shows a woman purchasing a radio at a store.)

9. How can Mrs. Taylor pay $30?
10. How much money will she have left? (Real Math, Grade 3, p. 25)

Like Addison-Wesley Mathematics, Real Math presents some problems that have more than one possible answer. For example:

Mr. Segal can finish a 1-kilometer race in 3 minutes. How long do you think it would take him to finish a 10-kilometer race? (Real Math, Grade 3, p. 264)

In contrast to Addison-Wesley's approach of taking the students step by step through a convergent strategy for calculating the answer, as the previous example of an Addison-Wesley word problem shows, Real Math leaves open the question of which strategies to use and which factors to consider. Another contrast to Addison-Wesley found in these "answers will vary" problems is the emphasis placed on finding a reasonable answer. The Real Math teacher's guide suggests, "Accept all reasonable answers greater than 30 minutes" (Grade 5, p. 264). Unlike the Addison-Wesley question, to which there were "no right answers," the reasonableness of one's answer is a significant consideration. In other words, there are some answers that definitely do not make sense.

Thinking Stories. Word problems only represent one type of the problems found in Real Math. Problem situations are also posed in the form of Thinking Stories. These are brief stories which present interesting, problematic situations, followed by questions to discuss. They are read aloud to the students in first through third grade and are contained in the student text in the upper grades. Additional problematic situations are presented throughout the fourth- through sixth-grade texts. Many are designed to generate thought and discussion and cannot be solved algorithmically. Focusing on mathematical ideas in realistic contexts, they call for thoughtful consideration, strategy development, discussion and sense making, as well as knowledge of the meaning and procedure of mathematical operations.

The second-grade Thinking Story Weighing Bowser is an example of a problematic situation designed to initiate discussion-exploring concepts related to measuring weight. In the story a group of children are trying to weigh a dog, Bowser. As they struggle to keep him on the scale they discover that "the harder you press to hold him down, the more he seems to weigh." Through questioning, the teacher is encouraged to lead the students in a discussion about why this is occurring and what they could do to avoid it.

Real Math uses problem situations to set the scene for instruction. The implicit message is that mathematical procedures and conventions have meaning and purpose and are not just rules given by the teacher to be followed by the students. Lesson five in fifth grade introduces the use of parentheses in symbolic statements with a brief story in which a cook wants to determine how many cookies to make for a group of three boys and two girls if he were to make five cookies per child. He asks the children, "How much is 5 x 3 + 2?" Three of the children answer seventeen; two say twenty five. Discussion questions which follow ask the students to speculate how the two different answers were calculated. After having students suggest ways to phrase the questions which will clearly ask for the number of cookies needed to supply three boys and two girls five cookies each, the statements (5 x 3) + 2 and 5 x (3 + 2) are introduced. Other Thinking Stories require students to follow clues to solve a mystery, decipher mathematical

codes, consider an infinite series, determine how much per month it costs to run a T-shirt store, and use dates to determine an average.

Discussion questions. Discussion questions, found in most lessons in the fourth- and sixth-grade texts, also encourage exploring and hypothesizing about intriguing mathematical ideas. The questions are usually intermixed with other parts of the lessons and are intended to be discussed by the entire class. The teacher's guide explains the main points of the question and suggests reasonable answers as well as answers the students might give. These questions can potentially contribute meaning to the mathematical concepts and skills being taught as students are required to reason about the ideas underlying them. For example, a fifth-grade lesson on averaging includes discussion questions that ask what information is omitted when a set of numbers are averaged and reduced to one number. Other parts of the lesson deal with the meaning of averages, as well as the mechanical steps for calculating them.

It is interesting to note that discussion questions are not included in the primary-grade texts. The fact that there are plenty of opportunities for discussion in the Thinking Stories in the primary grades indicates that the authors believe that younger children can discuss mathematical ideas. Thus, the absence of discussion questions at this level is all the more curious. It could be construed as a statement about primary content being perhaps fundamental or basic, something that must be established before more open-ended questions can be explored. In contrast to the upper grades texts, the content in the primary-grade texts reflects more traditional notions about essential mathematical knowledge, as it places greater emphasis on developing proficiency with basic operations. Another explanation for the difference between the primary- and upper grade texts might be the assumed reading level. The fourth-grade text makes an enormous leap from very limited reading requirements in the primary grades to a heavy emphasis on reading in the upper grades. Both the Thinking Stories and the brief problematic situations are found in the student text at this level, while the primary grade teachers read the Thinking Stories aloud to the class. Thus, what defines an appropriate problem could be somewhat determined by the student's reading level. This would suggest, however, that there would be more latitude in the

types of problems and questions included in the lower grades because they are read aloud by the teacher. It is indeed surprising that discussion questions are not part of the teacher's guide in the primary-grade texts.

The evidence provided by the word problems, Thinking Stories, and the consistent emphasis on discussion questions demonstrates the wide variety of problem-solving situations in Real Math. The focus is on making sense of situations and applying appropriate operations and strategies to find solutions. It also includes contemplating and discussing mathematical ideas with others. Similar to the suggestions in the Standards, the problems are often complex and messy, requiring the students to consider mathematical ideas which grow out of lifelike situations. Real Math does not include problems that grow out of mathematical situations or problems that are not placed in a lifelike context.

In Real Math, problem solving is balanced with direct skill development and computation practices. Problems are often used to set the scene for more direct instruction, rather than being the context in which skills and concepts are developed. The Thinking Stories are an exception to this, as they present problems which provide the contexts for developing and applying reasoning ability and mathematical skills. There is the possibility that these problems, since they are scattered throughout the curriculum, could be interpreted as enrichment activities that should be pursued once the rest of the chapter has been "mastered." According to the authors, all students are expected to engage in these problems.

Believing that students must understand the relevance of mathematical problems in their lives in order to be motivated to solve them, the Real Math authors attempt to place all problematic situations in some sort of lifelike context. While NCTM would agree that emphasizing the relevance of mathematics is crucial, the Standards also suggest that problematic situations which grow out of mathematical ideas have equal value in the elementary curriculum. Examples of this sort of problem are found in the Comprehensive School Mathematics Program.

Comprehensive School Mathematics Program

The development of CSMP grew largely out of dissatisfaction with the accepted content and pedagogical practices of commonly used mathematics curricula of the late 1960's. (Funded by the National Institute of Education, project development and testing began in 1966 and continued until the final versions were written and tested between 1978 and 1983.) One area of weakness identified was the lack of attention given to problem solving. The developers believed that problem solving was difficult to develop and teachers were generally underprepared to teach it. As a result, the development of problem-solving abilities became a central focus of the CSMP curriculum, and consequently of each lesson (Herbert, 1984).

Problem situations. As used in CSMP, the term "problem" refers to problematic and interesting situations which form the contexts through which students explore mathematical ideas as they work toward solutions. Some are situations one may be likely to meet in real life; others are based on fantasy, unusual events, or mathematical ideas. The purpose, however, is not to create lifelike situations to which mathematical procedures and skills are applied, as seen in Addison-Wesley Mathematics and some of the Real Math word problems. It is to create contexts which interest students and facilitate the use and development of conceptual understanding, skills, and procedures. The belief that for young children fantasy is part of real life and that through fantastic situations students can find meaning and learn is similar to that espoused by the authors of Real Math, as demonstrated by their Thinking Stories. The CSMP authors, however, take the idea of using situations as contexts for learning further by shaping each lesson around these problems, rather than adding them onto lessons which teach specific skills. The authors refer to their use of a situational problem as the central focus of each lesson as a "pedagogy of situations." The intent is for students to learn as they react to, explore, make sense of, and develop strategies to work within these situations.

The problem situations posed in CSMP grow out of mathematical ideas as they are taken from real life or fantasy situations. Such mathematically based problems are similar to those called for by the Standards. While NCTM recommends taking most problems from real-life experiences

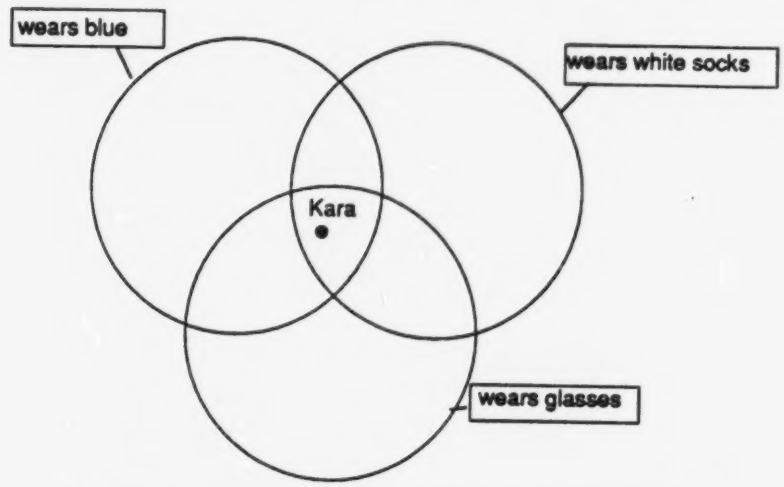
in the primary grades to ensure that the students learn about the relevance of mathematics in other situations, they recommend that mathematically based problems be added as the students progress. In contrast to NCTM's suggestion that by fifth grade the curriculum should contain a balance of problems arising out of empirical situations and mathematical ideas, CSMP seeks this balance as early as third grade. In the lower primary grades the problem situations consist primarily of fantasy situations. For example, a peanut-collecting elephant's encounter with magic peanuts introduces the concept of negative numbers at the first grade level. (Every magic peanut joins with a regular peanut and they both disappear.)

The emphasis in CSMP on whole-class lessons is not reflective of a belief that all students should be at the same level at one time. Rather, it is based on the authors' pedagogical assumption that students at various levels can contribute to and learn from a rich collaborative, problem-solving effort in different ways (Intermediate Grades, Part III, General Introduction). All students, not just the brightest, the authors argue, should have access to big mathematical ideas. In contrast to the common mastery approach, the program follows a spiraling pattern, returning frequently to various concepts. This approach assumes that each student will continue to build upon her understanding of that concept each time the topic arises, while continually integrating these ideas with others being explored in surrounding lessons.

Applications. This program uses mathematically based and real-life or fantasy-based problems as a site for applying mathematical thinking and reasoning. Conventional skills and procedures are seen as tools for solving problems and their development is integrated into the solution procedures. This is unlike more common interpretations of application which refer to computational skills being applied to lifelike situations in the form of story problems. CSMP's intention seems to be to have students apply their mathematical reasoning about the world to a variety of problems.

Mathematically based problems. Venn diagrams, called string pictures, are used to present problems of classification. Early introductions to string pictures have students classify objects and people according to various characteristics or attributes familiar to the students. A

lesson very early in the second-grade curriculum, for example, suggests that the teacher select labels for three overlapping strings based on the students' attire with the intention of determining labels so that at least one student is represented in each of the eight regions.



After explaining the attribute for each string the teacher presents the problem of determining where on the diagram several students belong. A dot is added to the diagram with a child's name beside it for each student placed. Once several students' names are on the diagram and most regions (including that outside of all three strings) have been represented another problem is presented. The teacher draws a dot on the diagram explaining that she is thinking of a student in the class and that is the dot for that person. The authors encourage that any name given (or region in the earlier part of the lesson) be scrutinized by the class as a whole, so that the group agrees with the suggested placement of each student. It is suggested that the class use evidence they have to determine whether or not Kara, for example, is indeed not wearing blue, wearing white socks, and glasses, before going on to the next suggestion. This use of class knowledge and evidence to determine the appropriateness of problem solutions is found throughout the CSMP curriculum, making it unlike Open Court and Addison-Wesley, both of which place the authority for knowing on the teacher and the text.

Several lessons later the students use string pictures to classify blocks of various shapes, colors, and sizes. Red, yellow, blue, and green circles, squares, and triangles, half of which are large, half small, are used in a lesson in a similar format. As the students progress in their ability to find relationships and determine and classify attributes, the problems presented become more complex. Another string game played throughout the curriculum begins with intersecting strings, the labels of which are face down. The problem is to determine the label of each string. As students take turns placing blocks in a region, the teacher indicates whether or not it belongs according to the labels. If it belongs, the block remains in the region. If not, it is returned to the pool of blocks. Both correct and incorrect placements act as clues to the students as they reason about the hidden labels of each string by noting which blocks belong and which do not.

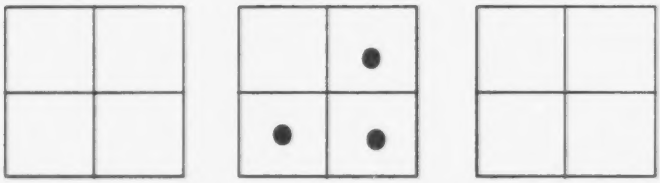
String games are also used to explore number theory. In these cases, number characteristics (e.g., odd numbers, numbers greater than 50, multiples of 8, divisors of 36) are used as string labels. The numbers to be placed in the strings can be written in various ways (e.g., 36, 6 x 6, 40 - 4, 72 ÷ 2, etc.). Labels that identify negations of attributes (e.g., not yellow or not multiples of 5) pose problems requiring students to think about the characteristics of a particular set and then about those items that are not members of the set.

Problems involving set theory or the mathematics of sets are found throughout the CSMP curriculum, each building on ideas developed in previous lessons. While these problems are not practical in that instances where they can be directly applied cannot be found in daily life, the authors argue that experiences with such ideas have a place in the elementary curriculum because they "can help the students to understand and use the ideas of classification. The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity, in general, and for the pursuit and understanding of mathematics in particular" (Upper Primary Grades, Part I, Language of Strings and Arrows, p. v).

The string picture is one of the three, essentially nonverbal, representational languages used by CSMP to set mathematical problems and communicate ideas and relationships without

prerequisite language. The Papy Minicomputer (a paper abacus on which numbers can be represented and mathematical operations performed) and arrow pictures (colored arrows connecting dots) represent relationships between numbers and objects; they are also visual languages introduced in first grade and built upon throughout the curriculum. Below are examples of how the minicomputer and arrow roads can be used.

The minicomputer



The minicomputer is one of the three visual representations used in CSMP. One of its strengths is its representation of base-10 place value. There is one board for each place, ones, tens, hundreds, etc. Numbers are shown on the board by placing magnetic checkers on the individual colored squares (which have the values shown below) and summing their value.

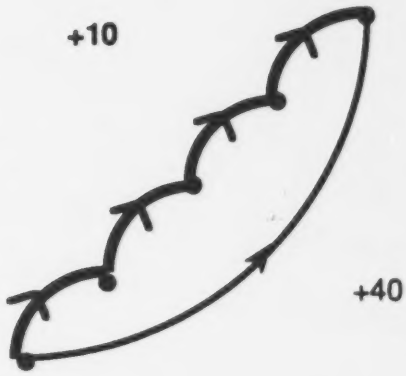
8	4
2	1

For example, a checker in the 2 square and one on the 4 square show 6. The same configuration on the tens board shows 60. The above boards show the number 70; there is a checker on the 4, 2, and 1 squares on the board in the tens place, showing that 40 + 20 + 10 = 70. A variety of other configurations could also show 70. The same configuration on the ones board would represent the number 7.

Computations are done on the boards by "making plays," which means moving and exchanging checkers while keeping the value of the number the same. Two checkers on the 4 square can be exchanged for one checker on the 8 square, as two 4s are the same as 8. To make a play from the ones board to tens board a checker from the 2 square and from the 8 square are moved to the 10 square on the next board, as 8 and 2 is the same as 10. Backward plays are similar. Since 40 is the same as two 20s, a checker on the 40 square can be exchanged for two checkers on the 20 square. To take half of a number, backwards plays must be made until there are exactly two checkers on each square. Then half of the checkers, one from each square, are removed. Multiplication is represented as repeated addition. Specially marked checkers are used to represent negative numbers.

Arrow roads

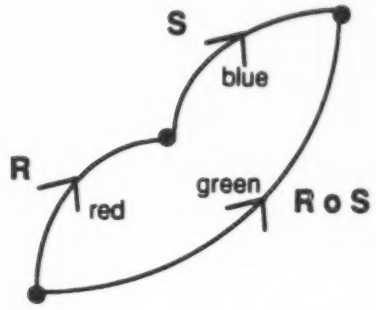
Arrow roads are colored arrows connecting dots which stand for numbers or objects. Each arrow represents a different function. This arrow road is made up of four +10 functions and one +40 functions.



Arrowroads can stand for other arithmetic or nonarithmetic relations, such as 5x, +4, or $\frac{1}{2}$ x (read half of), or you are my mother, I sent a letter to you, I am older than you.

These languages are unique in that they communicate ideas that the authors feel are central to mathematical thinking (classifying, recognizing functions and relations, flexibly manipulating numbers) and provide access to mathematical concepts before standard vocabulary and notations are introduced. This differs from the way representations in Addison-Wesley Mathematics and Real Math are used--as tools in finding solutions or answers, or to model the concept underlying a mathematical procedure, or the procedure itself. Sticks and bundles, for example, are used to concretely represent ones and tens when teaching students about place value and in regrouping to add and subtract. The representations in CSMP model larger mathematical ideas out of which specific concepts and procedures have developed, and they are used to pose problems within the domain of these ideas.

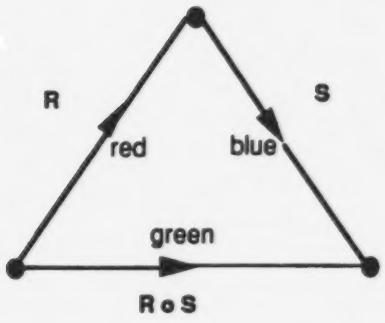
A fifth-grade lesson uses colored arrow pictures to develop the idea of composition of relations, the consistent relationship that is formed by composing two or more relations. Each relation is represented by a colored arrow, either red or blue, and the relation formed by the two is represented by a green arrow. Using the following picture, the teacher explains that a red arrow (R, written in red) followed by a blue arrow (S written in blue) can be represented by a green arrow (R o S written in green).



R o S, a red arrow followed by a blue arrow is represented by a green arrow.

After discussing possible compositions (see above) the students are given the problem of finding and drawing all possible green arrows in a complex drawing of red and blue arrows. Later in the lesson the students explore the different meaning the arrows could have and the following chart is developed (see Figure 4). The number of possible meanings the arrows can have is infinite. While there are certainly correct solutions, there are multiple and often unlimited possibilities. Whereas representations in Addison-Wesley Mathematics and Real Math tend to be selected to model a specific mathematical concept, the representations in CSMP cut across the entire curriculum representing a broad range of mathematical relationships.

Real-life and fantasy-based problems. Although many of the problems posed throughout the CSMP curriculum are set in mathematical contexts like those described above, nonmathematical settings, both lifelike and fantasy, also provide contexts for problems. Some of the lifelike application problems (the authors call them practical problems) include comparing money to decimals, dividing quantities into equal parts and determining the probability of actual events. A second-grade lesson employs a practical situation to pose a problem that involves the addition of decimal numbers. The lesson begins with the following story: "Colin has one dollar to spend in a whistle shop. He especially likes six of the whistles at this shop." (It is recommended that the child's name and type of shop be altered to be made appropriate for the particular class.) The teacher then draws a large oval on the board with six dots inside, each representing one whistle. One of the following prices is written next to each dot and the students are encouraged

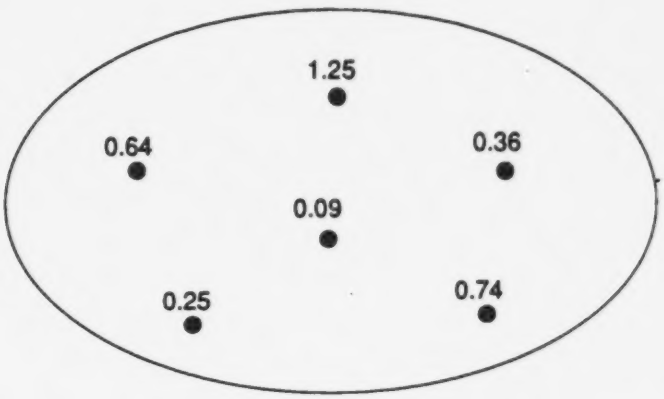


R	S	R o S
+ 5	- 8	- 3
you are my father	you are my mother	you are my paternal grandmother
- 4	- 7	- 11
2x	3x	6x
you are in my math class	you are not in my math class	you are not in my math class
4x	$\frac{1}{2}^{x}$	2x
you are my father	you are my sister	you are my aunt

The meaning of the green arrow (R o S) depends on the meaning of R and S. For example, if the red arrow represents +5 and the blue represents -8, the green would express the relation -3. If the red arrow indicates that the name at the end of the arrow is the father of the person written at the beginning, the person at the end of the green arrow is the paternal grandmother of the person at the beginning.

Figure 4. A chart is used to show the constant relationship among the three arrows.

to read the numbers and say how many cents each whistle costs: 0.09, 0.25, 0.64, 1.25, 0.36, and 0.74:



The class notes which whistle is the cheapest, which is the most expensive, and which the child cannot afford and why. Then the teacher poses this situation: "After thinking a while, Colin found that he could buy two whistles and spend all of his money. Which two whistles are they?" The authors suggest experimenting with different combinations of whistles suggested by the students. The students are encouraged to estimate in the process of selecting reasonable solutions, ruling out those that are obviously impossible. After a student suggests two possible prices, it is recommended that the teacher ask her to add them and explain how she calculated the answer.

Once the class solves the first problem a similar situation is presented with different prices and a new amount of money spent. This time it is suggested that the students be encouraged to try to find the correct combination on their papers before making the suggestion to the class. The authors suggest that the teacher write suggested prices on the board and ask a student to do the calculation. The notes in the teacher's guide tell which answers might be acceptable. This use of the students' problem-solving process as the substance of each mathematics lesson is a characteristic unique to CSMP.

Problem-solving strategies. Students' solution strategies are treated seriously. As in Real Math, it is expected that the students will develop and use strategies as they work through

the problem situations. The whole-class lessons facilitate strategy development within a community. As the class works to solve each problem, it is suggested that each possible solution be scrutinized by the class before being accepted as one possible answer.

As the class progresses through the curriculum, ideas from previous lessons are taken up and extended in new lesson situations. As explained above, negative numbers are introduced through the story of Eli the elephant who collects regular and magic peanuts. He discovers that one regular peanut cancels out one magic peanut. Students use this to cancel peanuts in Eli's bag and to make their own bags contain certain numbers of peanuts, using various combinations of magic and regular peanuts. New problem situations are used throughout the curriculum to build on the concept of negative numbers in order to encourage different ways of reasoning about this concept and to provide students with new opportunities to develop solution strategies. In second grade the students are given the problem of writing a number sentence which expresses what is happening in Eli's bag. Negative numbers then begin to show up in the minicomputer and arrow-picture problems. They are also located on the number line. In a third-grade lesson, a story about the various floors of the Empire State Building is used to model a number line. The ground floor is 0. The floors above the ground are labeled with positive numbers and those below are labeled with negative numbers. An elevator moves people up or down to various floors, but it only has a +1 and -1 button. The students begin working with the problem of how many floors and in which direction the elevator would travel to go from one designated floor to another. The problem is then complicated by pretending that the elevator has the following buttons: -5, +5, -1, and +10. The students strategize ways to get to certain floors with this restriction. Problems such as these provide the central focus of each CSMP lesson.

The authors of CSMP see problem solving as central to learning mathematics. They have designed lessons that grow out of problem situations, intending to provide a context for the development of mathematical understanding, skills, and procedures. The authors of Math in Stride have taken a similar stance on the role of problem solving in the curriculum and in each lesson. Like CSMP, the students confront problem situations which allow them to grapple with

mathematical ideas. In CSMP it is through pictorial representations or nonverbal languages; in Math in Stride it is through concrete materials. A distinct difference between the two is that Math in Stride in based on developmental theory while CSMP is based on a view of the discipline of mathematics.

Math in Stride

Math in Stride is an activity-based program published by Addison-Wesley, but quite unlike this publisher's commonly used curriculum. It focuses on the child's development of mathematical understanding through interaction with her environment. Based on the Piagetian theory that a child's individual, conceptual development is nurtured through guided experience interacting with and making sense of her environment, Math in Stride is designed to engage the student in situations which pose quantitative, spatial, and organizational problems. Claiming that each child goes through similar stages of intellectual development, but in very different ways, the authors have designed activities for the individual or small group. This is unlike CSMP, which places whole-class problem solving as central to the development of mathematical understanding in the classroom and consequently within the individual.

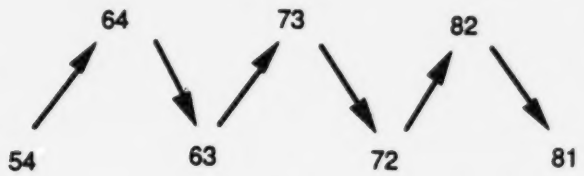
Problem situations. Similar to CSMP, each Math in Stride lesson is organized around a problem situation. The problems are posed within the context of concrete objects or pictorial representations with which the students interact and experiment to reach solutions. The experiences the students are intended to have through interaction with manipulatives or pictures include exploration, invention, experimentation, calculation, developing and using strategies, discovering relationships and recording and analyzing data. A typical lesson begins with the teacher presenting a problem or activity to the whole class or a small group. The students then spend the bulk of each lesson interacting with materials or other representations, often recording their findings. This is usually followed by a discussion of the findings or a worksheet which presents an extension of the previous activity.

A second grade geometry lesson involving exploration with geoboards, for example, suggests that the teacher work with a small group of students. Once they have experimented with one-band designs (creating shapes on the geoboard with a single rubber band) the teacher should create shapes--such as horizontal, vertical, or diagonal lines as well as regular and irregular geometric shapes--that the students try to replicate on their boards. Next, students should be encouraged to create their own designs that others observe and copy. While these activities are taking place, it is suggested that the teacher and the students discuss different properties of their designs. Lessons following this one have the students construct designs with multiple rubber bands and record them by copying them on to dot paper, reproduce designs and shapes on dot paper on their boards, and compare various designs attending to size, shape, and position. In later lessons students will use geoboards as well as other concrete materials to estimate and calculate the area and perimeter of various shapes.

Application. The conception of application of mathematics in Math in Stride is similar to that in CSMP in that it emphasizes applying mathematical thinking and resources--knowledge of procedures and strategies (Schoenfeld, 1985; Putnam, Lampert, & Peterson, 1989)--to solving problems. The authors group these problems into two categories: translation problems and process problems.

Process problems describe the majority of problems found in Math in Stride and are problem situations requiring students to develop and use strategies and thinking processes in order to solve them. Specifically, the authors intend students to develop abilities in observing, hypothesizing, recording and analyzing data, and drawing conclusions that can be applied to other problem situations.

An example fifth-grade lesson presents sequences of numbers such as:



(Grade 5, p. 32)

Students are asked to discover the rule of each pattern following the arrows (+10 then -1) and following the numbers across horizontally (+9). The lesson goes on to have students extend the sequences and to create their own, discussing the strategies they used. This problem is not unlike problems in CSMP which use arrowroads to represent the composition of functions. It is atypical for the Math in Stride program in that it is designed as a whole-class lesson and does not use concrete materials. These characteristics are more common, however, to the upper elementary grades than the primary level.

Problems like these provide contexts for algorithmic skills to be developed and practiced. This is the reverse of more traditional approaches that introduce the computational skills and then use word problems as sites for their application, such as in Addison-Wesley Mathematics.

Occurring less frequently than process problems, translation problems are merely problems translated into symbolic statements. These problems vary from typical word problems set alongside or to be matched with corresponding symbolic statements to symbolic statements or general information from which word problems are to be written. Below are some examples of translation problems from grade five:

Keneesha sold 8 ornaments for $.49. How much money did she collect?

10 x $.49 - $1.00 - $.02 =
8 x $.50 - $.08 =
4 x $1.00 - $.08 =
$.49 - $.08 =
8 x $.49 =

Instructions to student: solve each equation. Circle equations that fit the story. (p. 45)

Write questions and equations for each story below. Complete the equation.

Eddie went to the grocery store to buy oranges for his mother. They cost $.98 for three. He had a five-dollar bill. (p. 47)

Make up a story from the information given. Then write questions about the story. Write and solve equations for the question.

1975
Bobby is six years old.
Anna is ten years old.
Yung-Su was five years old in 1972. (p. 48)

As seen in the example, many of the problems have multiple solutions and require the solver to understand the situation and the symbolic statement.

Strategy development. The Math in Stride program does not teach specific problem-solving strategies. The stance that strategy development is encouraged and intended is reflected in the teacher's guide which suggests that students experiment with various strategies, discuss and compare them, and do activities several times trying alternative approaches. A second-grade lesson, for example, suggests that students use tangram pieces to match shapes on a work-sheet page and then use matching shapes to build the other half onto the original shape. These halves can be added at various places and it is recommended that the teacher encourage experimenting with different ways to build on the half. (See Figure 5.)

Like CSMP, Math in Stride uses problems as contexts for learning and exploring mathematics. The use of Piagetian theory of development to guide the curricular goals and sequencing is unique to Math in Stride.

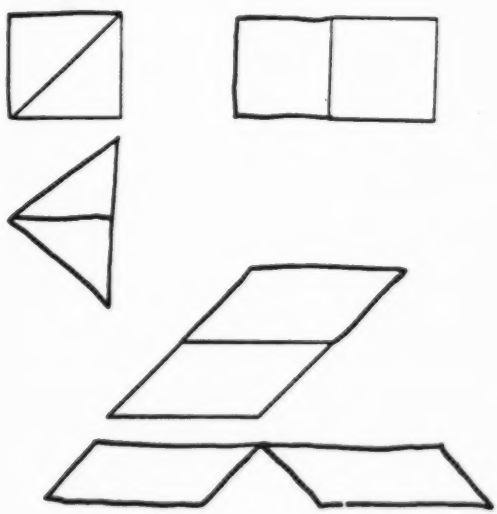
A Comparative Look at the Four Curricula

The attention given to problem solving in these four curricula, both rhetorically and substantively, illustrates that the calls for attention to developing mathematical problem-solving abilities have been, in some way, heard. It has influenced the mathematics education community at all levels. Both commonly used and distinctive or alternative curricula give it top billing. For

Name _____

Use Tangram pieces. Cover the shape.
Build the other half. Trace it. Color.

(Answers will vary. Some possible answers are shown.)



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(Answers will vary.) _____

Use Tangram pieces. Cover the shape.
Build the other half. Trace it. Color.

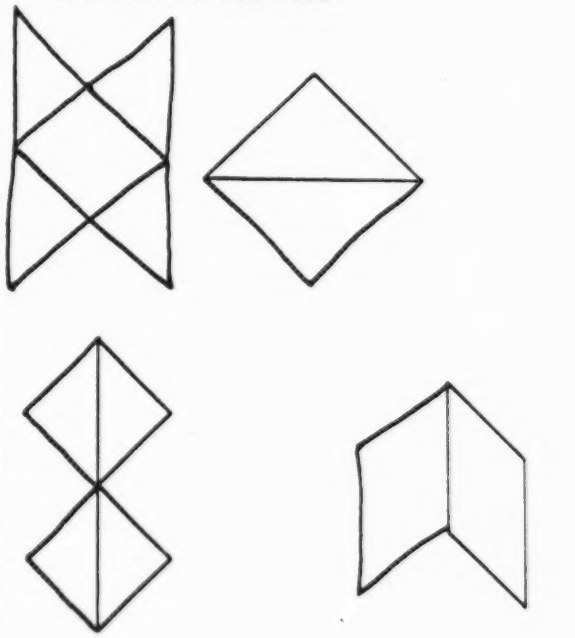

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Figure 5. Building on the other half, Math in Stride (Addison-Wesley Innovative Division, 1988, Teacher's Edition, Book 2, p. 202).

some alternative curricula, CSMP for example, problem solving is not new. Yet the fact that it has taken hold in programs ranging from alternative to commonly used suggests that awareness of its necessity is at a new height. Indeed, this concern is reflected in the research community which has increased its attention to mathematical problem solving over the last five years (Schoenfeld, 1985; Charles & Silver, 1988). The NCTM Standards bolster these efforts by describing mathematics as problem solving.

Analyses of the problem solving found in these four elementary curricula--Addison-Wesley Mathematics, Real Math, Comprehensive School Mathematics Program,and Math in Stride--reveal strikingly different conceptions of what real problems are and are like, their place in the curriculum, and how problem solving is learned or developed. Comparing these various perspectives on problem solving, the levels at which it is incorporated into the curriculum and how, indicates, perhaps, the degree to which the authors of each view "mathematics as problem solving."

Real Mathematical Problems

The Standards differentiate between neat, traditional word problems in which algorithmic exercises are nested, and complicated or multiple-step nonroutine problems. This distinction is significant since "real-world problems are not ready-made exercises with easily processed procedures and numbers" (National Council of Teachers of Mathematics, 1989, p. 76). This distinction is illustrated by contrasting the single-step word problems in Addison-Wesley Mathematics and Real Math to the more complex problems which set the context of CSMP and Math in Stride lessons. The terms "translation problems" and "process problems" are used by the authors of Math in Stride to delineate between these two types of problems. Comparing the straightforward translation problems found in Addison-Wesley Mathematics to those that require the solver to make sense of the situation in Real Math and Math in Stride suggests that the labels "routine" or " translation" leaves much of the story untold.

A deeper level of analysis of these problems, one that considers the context of the problem situation, the relevance of the mathematical task to the problem situation, and the actual value of the mathematical task, reveals more subtle differences between the problems in these four curricula. While Addison-Wesley Mathematics and Real Math limit the problem situations used to lifelike or fantasy situations or experiences, CSMP and Math in Stride draw from mathematics and empirical problems, in addition to real and imaginary life situations, in formulating problems. Unlike the word problems in Addison-Wesley Mathematics which often contain mathematical tasks that are tangential to the situation, those in Real Math contain tasks that grow naturally out of the situation and are less contrived.

When considering the value of the mathematical tasks posed in the problems we find that some word problems come closer to being complex and messy than others, while seemingly nonroutine problems can propose a meaningless or unproblematic task. Although the Standards suggest that word problems are straightforward or neat, this analysis found that the degree to which a problem had a simple, known solution rather than being problematic was determined by the nature of the task posed by the problem and its treatment in the text. The previously cited Addison-Wesley problem, which asks the student what time she would leave home to get to a movie on time is an example of a potentially nonroutine problem which has been simplified and transformed to a computational exercise. Word problems cited as examples from the three distinctive programs may have embedded, single-answer algorithms, yet the tasks include determining what the question is, selecting the appropriate operation and using it correctly. Although it is not the case in every instance, it appears that the three distinctive curricula propose more meaningful mathematical tasks that grow naturally out of their problem situations than does the commonly used text.

Purpose of Problem Solving

Another point of contrast between the four curricula is the purposes that problem solving has in each curriculum. That problem solving is seen as a form of application is explicitly stated in

Addison-Wesley Mathematics and is implied to varying degrees in the other three curricula. The variety of problem situations and their placement in the texts suggests different perspectives on what is being applied in these application problems. In Addison-Wesley Mathematics, the student applies computational skills or prescribed problem-solving strategies to straightforward problems. This is particularly salient in sequences in which skills are taught, practiced, and then placed within word problems to be applied. Real Math also uses word problems as sites for practicing computational skills. These word problems are integrated throughout the lessons and the required operations are mixed. The Thinking Stories in Real Math are similar to the problem situations in CSMP and Math in Stride in that mathematical thinking and reasoning is applied. Computational skills and conventions are tools used to solve the problem. The situations in Real Math focus on mathematical ideas in realistic experiences, the manipulatives in Math in Stride draw the students' attention to problems within the physical environment, while the pictorial languages of CSMP have students grapple with abstract mathematical ideas. In all cases, this sort of application assumes that the problematic situation will give rise to the development of strategies, concepts, and skills as the solvers work through the problem. A sequential, mastery approach, on the other hand, assumes that these skills must be explicitly taught before they can be applied.

Epistemological and Pedagogical Assumptions

Epistemologically and pedagogically the approaches to problem solving in the three distinctive curricula do provide alternatives to traditionally held views of mathematics. Perspectives that mathematics is a collection of rules and procedures that must be memorized and should be taught sequentially, in isolation from one another, pervade the mainstream society, inside and outside of schools (Ball, 1988). Addison-Wesley's sequential, step-by-step approach and prescriptive method of teaching problem-solving strategies does not break from these societal views. The authors of Real Math have managed to break from society's hold to a great degree, particularly through the problem situations presented in the Thinking Stories. Nevertheless, this curricula rests heavily on isolating and teaching algorithmic skills outside of

problem contexts. CSMP and Math in Stride, on the other hand, reject the mastery approach and view of mathematics as discrete pieces. Despite distinct theoretical underpinnings of the programs, both sets of authors have integrated computation and skill development into problem situations and emphasize that these skills are tools to be learned for the purpose of solving problems, rather than as ends in themselves.

Conclusion

The conceptions of problem solving found in these four curricula reflect views of mathematical problems and problem solving professed by various authors and publishers. They provide images of what problem solving could look like in the classroom. It must be kept in mind that these images and the degree to which they fit with the perspective portrayed in the Standards are incomplete because they are restricted to the printed page. What remains unknown is how they become enacted and used in the classroom as they are translated by teachers from written words to classroom activities. Together with the Standards, these curricula provide resources and guides for teachers. The three distinctive curricula offer resources that are alternatives to those most commonly used by mathematics teachers. Considering that the three alternative curricula present views of mathematics, problem solving, and how solutions are learned that break from mainstream perspectives, it is important to understand how teachers understand and use such resources. Because they propose alternative and perhaps new views on what problem solving is and on teaching children to become mathematical problem solvers, we have much to learn about how and in what ways they influence actual classroom curriculum.

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